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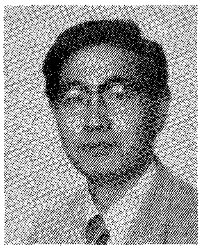

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Hybrid Modes in Circular Cylindrical Optical Fibers

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Abstract—**The classification of hybrid modes in circular cylindrical optical fibers is studied. It is shown that problems of the mode classifications, i.e., the crossover of the dispersion characteristics and the remarkable changes of the polarization states and the field configurations, are caused by the coupling of the HE-type and EH-type modes.**

I. Introduction

RECENTLY, various optical fibers, including multimode fibers, single-mode fibers, single polarization fibers, and dual-mode fibers, have been produced. In general, propagation modes supported by multimode fibers are called linearly polarized (LP) "modes" [1], which are not true modes. In case the optical fibers have few propagation modes, the difference between the corresponding true modes for each LP "mode" is of practical importance for calculating fiber bandwidths and designing fibers [2]. The descriptive names TE, TM, HE, and EH are usually used for distinguishing propagation modes. There are several classifications of hybrid modes, which are based on the amplitude coefficient ratio of axial components of electric and magnetic fields [3], [4], the polarization states [5], [6] and the field configurations [7] of propagation modes, and the factorization of characteristic equations [8], [9].

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Safaai-Jazi and Yip [8] analytically showed that the amplitude coefficient ratio of axial components cannot be utilized for the designation of hybrid modes for three-layer cylindrical dielectric waveguides and no "crossover," as has been often observed [4], [10], [11], exists between hybrid modes with the same order of azimuthal variation. Hashimoto [12], [13] mentioned that the classifications based on the amplitude coefficient ratio and the polarization states have disadvantages for certain classes of refractive-index profile. It is the purpose of this paper to clarify the polarization states and the field configurations of hybrid modes at the vicinity of the crossover, to show the reason why crossover is caused by most mode classifications, and to propose a new scheme for the classification of hybrid modes in cylindrical optical fibers.

II. Various Mode Classifications

Several classifications of hybrid modes are described briefly in this section. Snitzer [3] suggested a scheme based on the value of the amplitude coefficient ratio of axial components of electric and magnetic fields

$$P = -\frac{\omega\mu_0}{\beta}\frac{H_z}{E_z} \tag{1}$$

where β is the propagation constant of the guided mode, ω is an angular frequency, and μ_0 is the free-space permeability. The modes for which P is positive are designated as EH, and those for which it is negative as HE. Kuhn [4] has attempted to use the sign of this ratio to classify hybrid modes in a homogeneous fiber with finite cladding. In this paper, Snitzer's criterion is used for graded-index fibers by using the multilayer approximation method [2]. The coefficient ratio P is evaluated at the center layer.

Tanaka and Suematsu [9] proposed a mode designation based on the multilayer approximation of the refractive-index profile and the approximate factorization of characteristic equations. The electromagnetic fields in each layer are represented rigorously in terms of Bessel functions. The continuity of the tangential fields and the discontinuity of the normal fields at the boundary between ith and $(i+1)$st layers lead to the following relation:

$$\begin{bmatrix}\Phi_z^-\\ E^-\\ \Phi_z^+\\ E^+\end{bmatrix}_{r=a_{i+1}} = \begin{bmatrix} P_i^- & \begin{matrix}0 & 0\\ 0 & 0\end{matrix}\\ \begin{matrix}0 & 0\\ 0 & 0\end{matrix} & P_i^+\end{bmatrix} \cdot \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1+\Delta_i & 0 & \Delta_i\\ 0 & 0 & 1 & 0\\ 0 & \Delta_i & 0 & 1+\Delta_i\end{bmatrix}\begin{bmatrix}\Phi_z^-\\ E^-\\ \Phi_z^+\\ E^+\end{bmatrix}_{r=a_i} \tag{2}$$

where

$$\Phi_z^{\pm} = jE_z \pm \frac{\omega\mu_0}{\beta}H_z$$

$$E^{\pm} = E_r \pm jE_\theta$$

$$\Delta_i = \frac{n_{i-1}^2 - n_i^2}{2n_i^2}$$

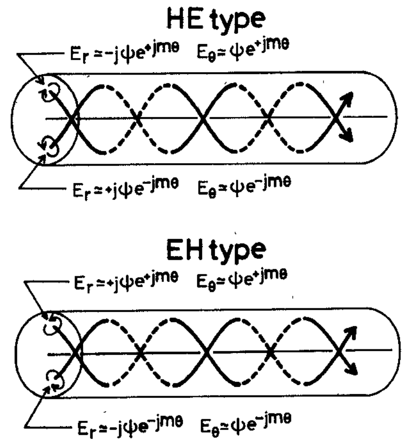

Fig. 1. Polarization states of hybrid modes in a dielectric rod.

and P_i^- and P_i^+ are 2×2 matrices constituted by Bessel functions. Eliminating Δ_i at off-diagonal elements, we can approximately separate (2) into two equations as follows:

$$\begin{bmatrix}\Phi_z^{\pm}\\ E^{\pm}\end{bmatrix}_{r=a_{i+1}} = P_i^{\pm}\begin{bmatrix}1 & 0\\ 0 & 1+\Delta_i\end{bmatrix}\begin{bmatrix}\Phi_z^{\pm}\\ E^{\pm}\end{bmatrix}_{r=a_i}. \tag{3}$$

The modes with negative and positive signs corresponds to the HE and EH modes, respectively. Approximate eigenvalue equations for the HE and EH modes are derived from (3). Hybrid modes in this classification will be distinguished from the exact solutions by a prime, i.e., HE′ and EH′.

The modal rays of the HE and EH modes in optical fibers are circularly polarized skew rays in opposite polarization states to each other [5], [6], as shown in Fig. 1. However, the polarization states and the field configurations of hybrid modes in certain classes of optical fibers change remarkably with increasing frequency. The designation of a mode should be determined by the characteristic curve, and not change with the frequency. To describe the polarization states and the field configurations of hybrid modes, we use the names HE_{mn}^r-type and EH_{mn}^r-type, whose polarization states and field configurations are similar to those of the HE_{mn} and EH_{mn} modes in the dielectric rod as a whole, respectively.

In this paper, the field distribution, the propagation constant, and the group velocity are rigorously (or exactly enough) obtained by means of a vector multilayer analysis [2]. In case the polarization states and the field configurations of hybrid modes vary markedly with the frequency, they are designated as Hyb_{mn} for no confusion of their properties. The first index m is related to the number of circumferential variations in the field. The second index n counts the order in which a particular Hyb_{mn} mode goes through cutoff with increasing frequency to become a guided mode of the fiber.

III. Hybrid Modes in Inhomogeneous Optical Fibers

In this section, the propagation constants, the group velocities, the field distributions, and the amplitude coefficient ratios of hybrid modes are calculated numerically for

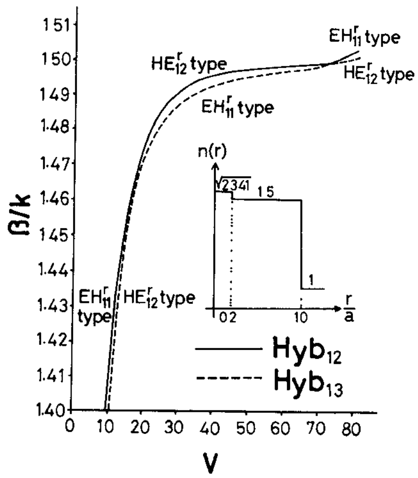

Fig. 2. Characteristics and the refractive-index distribution of a homogeneous fiber with finite cladding.

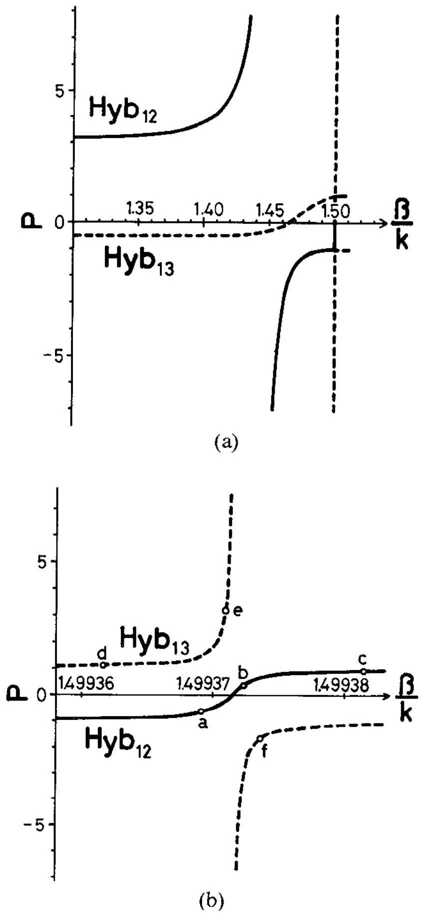

Fig. 3. Amplitude coefficient ratios for the Hyb_{12} and Hyb_{13} modes of a homogeneous fiber with finite cladding: (a) $\beta/k > 1.3$, (b) near $\beta/k = 1.5$.

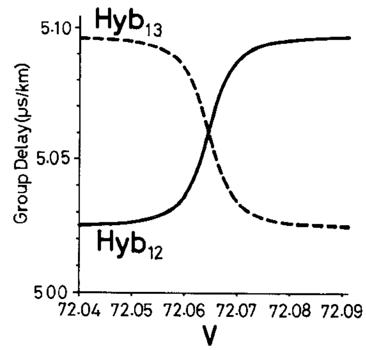

Fig. 4 Group delays for the Hyb_{12} and Hyb_{13} modes of a homogeneous fiber with finite cladding.

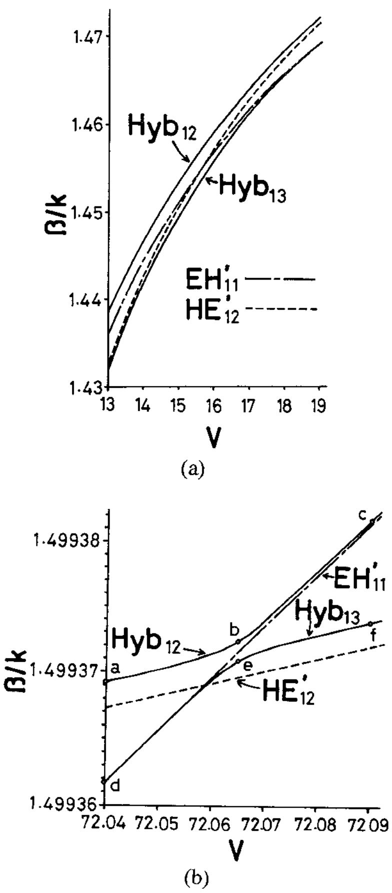

Fig. 5. Characteristics of a homogeneous fiber with finite cladding calculated by the approximate factorization method and the rigorous multilayer method: (a) near $V = 16$, (b) near $V = 72$.

three-layer cylindrical dielectric waveguides. The solutions obtained by the approximate factorization method are compared with the vector rigorous solutions, and several interesting phenomena are pointed out. It is shown that the phenomena occur for a square-law index fiber which is one of the typical graded-index fibers.

The normalized frequencies V and T, the normalized propagation constant B, and the relative index difference Δ are defined as

$$V = kan_{\max}\sqrt{2\Delta}$$

$$T^2 = 2k^2\int_0^\infty \{n(r)^2 - n_{\min}^2\}\, r\,dr$$

$$B = \frac{\beta - kn_{\min}}{k(n_{\max} - n_{\min})}$$

$$\Delta = \frac{n_{\max}^2 - n_{\min}^2}{2n_{\max}^2}$$

where $n_{\max}$ and $n_{\min}$ are the maximum and the minimum refractive indices, respectively.

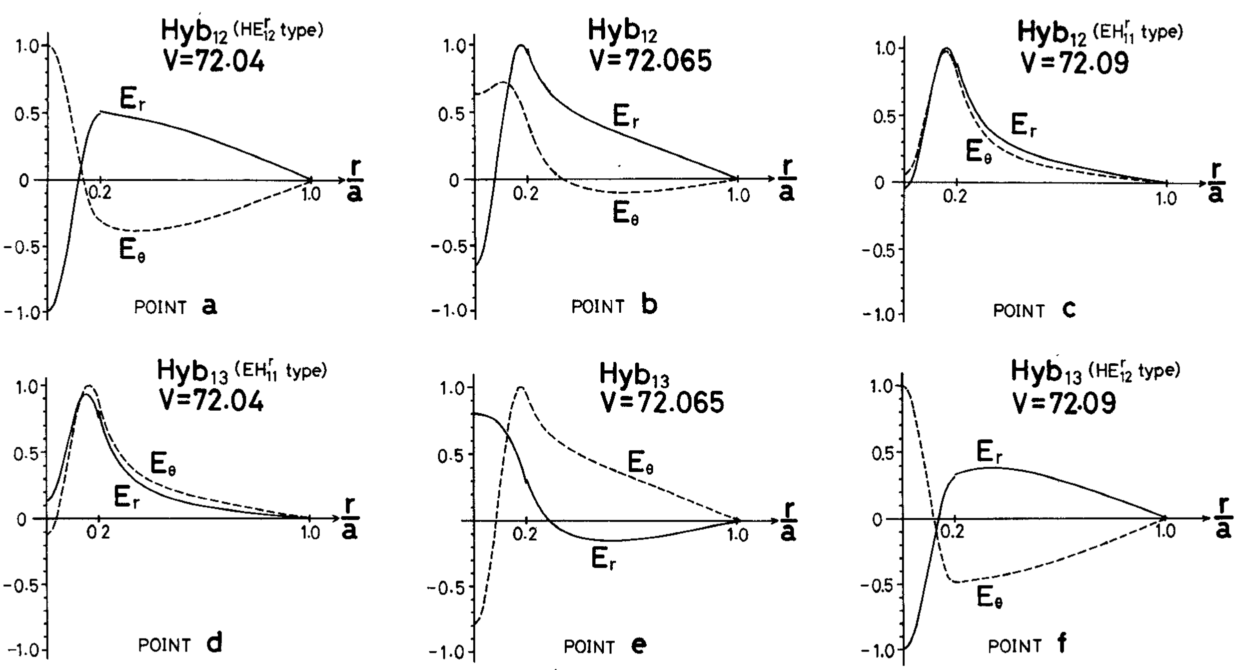

Fig. 6. Field distributions of the Hyb_{12} and Hyb_{13} modes in a homogeneous fiber with finite cladding at the points, a, b, c, d, e, and f, which are illustrated in Figs. 3(b) and 5(b).

A. Homogeneous Fiber with Finite Cladding

The dispersion characteristics and the refractive-index distribution of a homogeneous fiber with finite cladding are shown in Fig. 2. It is observed that the characteristics of the Hyb_{12} and Hyb_{13} modes, which are the EH^r_{11}-type and HE^r_{12}-type modes, get very close to each other near $V=16$ and 72, but do not cross each other. A similar phenomenon takes place between the EH^r_{mn}-type and HE^r_{mn+1}-type modes, which are not plotted in Fig. 2 for simplicity.

Both the Hyb_{12} and Hyb_{13} modes change signs of the amplitude coefficient ratios in the neighborhood of $V=16$ and 72, as shown in Fig. 3. Fig. 3(b) shows P at the vicinity of $V=72$ ($\beta/k=1.5$). We observe that P for the Hyb_{12} mode varies from $+\infty$ to $-\infty$ near $V=16$ and from -1 to $+1$ near $V=72$, and for the Hyb_{13} mode from negative to positive near $V=16$ and from $+\infty$ to $-\infty$ near $V=72$. Moreover, the Hyb_{12} and Hyb_{13} modes change the group delays with each other, as shown in Fig. 4. On the basis of this result, it may be impossible to use the amplitude coefficient ratio and the group delay to decide that the characteristics of hybrid modes with the same azimuthal number, the EH^r_{mn}-type and HE^r_{mn+1}-type modes, do not cross each other.

To investigate the phenomena near $V=16$ and 72 in detail, the homogeneous fiber with finite cladding is analyzed by using the approximate factorization method. Fig. 5 shows the dispersion characteristics calculated by the approximate factorization method and the rigorous vector multilayer method. It becomes evident that the characteristics of EH'_{11} and HE'_{12} cross each other where P changes sign.

Fig. 6 shows the field distributions of the Hyb_{12} and Hyb_{13} modes at the points, a, b, c, d, e, and f, which are given in Figs. 3(b) and 5(b). The field configuration of the Hyb_{12} mode at the point a is very different from that at the point c. The field distribution of the Hyb_{13} mode alters between the points d and f. Furthermore, Fig. 6(a) and (c) shows the same field distributions as Fig. 6(f) and (d), respectively. We find that the Hyb_{12} and Hyb_{13} modes change their field distributions with each other at the crossover between EH'_{11} and HE'_{12}. The symbols, EH^r_{11}-type and HE^r_{12}-type, are added in Fig. 2 to indicate the polarization states and the field configurations of the Hyb_{12} and Hyb_{13} modes.

At the crossover between EH'_{11} and HE'_{12}, leaving Δ_i in the off-diagonal elements leads to "coupling" between EH'_{11} and HE'_{12}. The coupling gives rise to the split of the characteristics, the changes of the polarization states, the field configurations, and the sign of P. In general, the characteristic curves cross each other between EH'_{mn} and HE'_{mn+1} for any number of m and n, and the coupling phenomenon takes place in the cladding mode region. The polarization states and the field configurations of EH'_{mn} and HE'_{mn+1} correspond to those of the EH^r_{mn}-type and HE^r_{mn+1}-type modes, respectively. The pair of hybrid modes, the EH^r_{mn}-type and HE^r_{mn+1}-type modes, interchange their polarization states and field configurations as the frequency increases. It becomes evident that the sign of P, the polarization states, and the field distributions cannot be utilized for the designation of hybrid modes in the cladding mode region. In the core mode region, however, they may be applicable to the designation of hybrid modes because their properties hold.

B. Dielectric Tube Waveguide

The refractive-index distribution and the dispersion characteristics of a dielectric tube waveguide are shown in Fig. 7. We find that no crossover exists between EH′ and

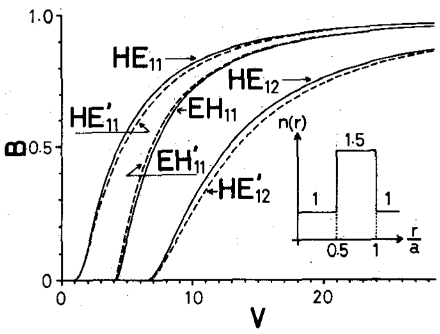

Fig. 7. Characteristics and the refractive-index distribution of a dielectric tube waveguide.

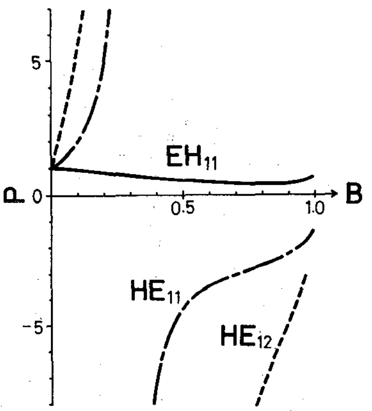

Fig. 8. Amplitude coefficient ratios of a dielectric tube waveguide.

HE′ modes with the same order of azimuthal variation. Therefore, the rigorous characteristics of hybrid modes do not get close to each other in the same way as those of the homogeneous fiber with finite cladding. The descriptive names HE and EH can be used without confusion of their polarization states and field configurations.

Fig. 8 shows the amplitude coefficient ratio P of a dielectric tube waveguide. The ratio P is always positive for the EH modes, while for the HE modes it varies from $+\infty$ to $-\infty$. On the basis of Figs. 7 and 8, no coupling, as is observed for the homogeneous fiber with finite cladding, occurs between hybrid modes. Therefore, hybrid modes hold their polarization states and field configurations stable. In the case of no coupling, we can classify the hybrid modes by the polarization states and the field configurations, i.e., the approximate factorization method.

C. Square-Law Index Fiber with Infinite Cladding

Such coupling phenomena as take place for the homogeneous fiber with finite cladding are observed for graded-index fibers. The field configurations, the amplitude coefficient ratios, and the propagation constants are calculated for a square-law index fiber. The permittivity distribution in a core region is represented approximately by 50 layers of different constant refractive indices.

Fig. 9 shows the refractive-index distribution and the amplitude coefficient ratios of a square-law index fiber. The ratio P varies from $+\infty$ to $-\infty$ for the Hyb_{13} mode, and from negative to positive for the Hyb_{12} mode near $B = 0.3858$, respectively. The sign of P, therefore, cannot

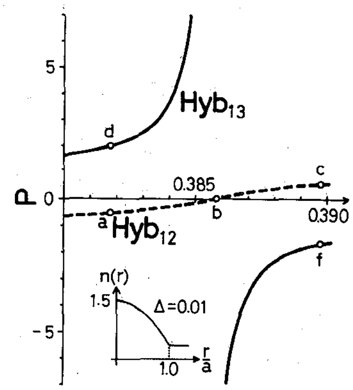

Fig. 9. Amplitude coefficient ratios and the refractive-index distribution of a square-law index fiber.

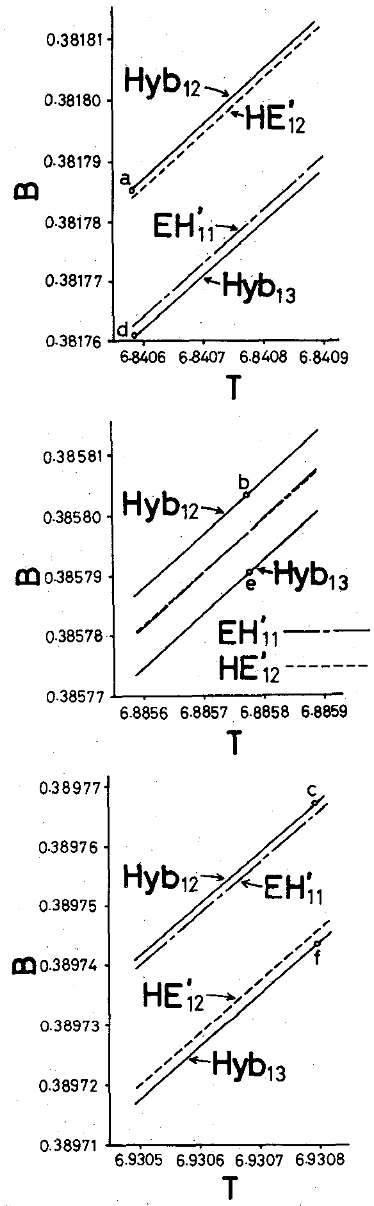

Fig. 10. Characteristics of a square-law index fiber calculated by the approximate factorization method and the rigorous multilayer method.

be utilized for the designation of hybrid modes.

The dispersion characteristics for the square-law index fiber are shown in Fig. 10. We observe[1] that the character-

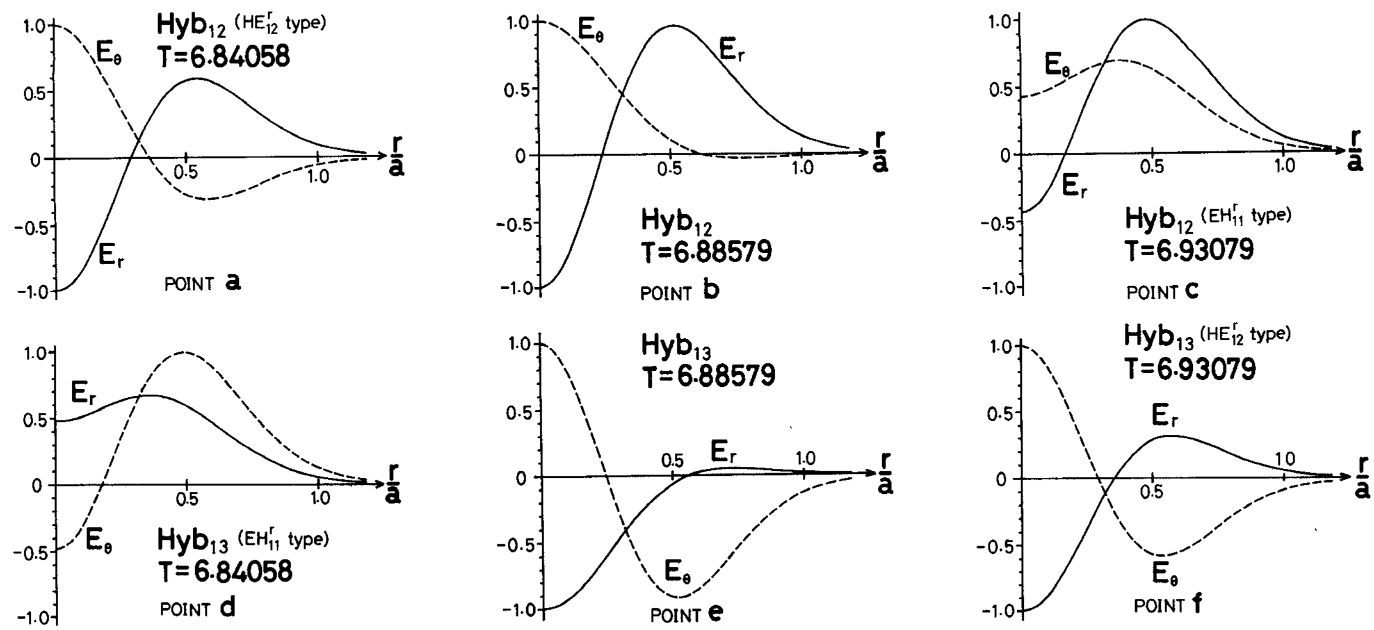

Fig. 11. Field distributions of the Hyb_{12} and Hyb_{13} modes in a square-law index fiber at the points, a, b, c, d, e, and f, which are illustrated in Figs. 9 and 10.

istics of the Hyb_{12} and Hyb_{13} modes get very close to each other, but do not cross each other. On the other hand, the crossover of the characteristics obtained by the approximate factorization method exists between EH'_{11} and HE'_{12}. In general, the crossover occurs between EH'_{mn} and HE'_{mn+1} for any number of m and n, which are not plotted in Fig. 10. The coupling phenomenon occurs between the EH^r_{mn}-type and HE^r_{mn+1}-type modes [14].

Fig. 11 shows the field distributions of the Hyb_{12} and Hyb_{13} modes at the points indicated in Figs. 9 and 10. The field distributions at the points a (Hyb_{12}) and f (Hyb_{13}) are HE^r_{12}-type, and those at the points c (Hyb_{12}) and d (Hyb_{13}) are EH^r_{11}-type. It can be concluded that the coupling causes the interchanges of the field configurations and the polarization states of the Hyb_{12} and Hyb_{13} modes. Therefore, the field configurations and the polarization states cannot be used for the designation of hybrid modes. In the case of the square-law index fiber, the coupling phenomenon happens in the core mode region, not in the cladding mode region.

IV. Discussion

Safaai-Jazi and Yip [8] analytically showed a mode classification based on the rigorous factorization of characteristic equations for three-layer cylindrical dielectric waveguides. However, the method cannot be used for general optical fibers because of difficulty in factorizing characteristic equations. Other methods, which are based on the amplitude coefficient ratio of axial components, the polarization states, the field configurations, and the approximate factorization of characteristic equations, are not applicable for the designation of hybrid modes, since they lead to the crossover of the dispersion characteristics as shown in the previous section.

[1]According to [12], [13], the characteristics of the EH^r_{mn}-type and HE^r_{mn+1}-type modes do not get very close to each other for a square-law index fiber *without* cladding.

It is generally believed that the HE_{m+1n} and EH_{m-1n} (or HE_{2n}, TM_{0n}, and TE_{0n}) modes are almost degenerate for weakly guiding fibers with similar transverse fields in the x and y-directions, and are equivalent to the LP_{mn} (or LP_{1n}) "mode" [15], [16]. In case the hybrid modes change their field distributions, as shown in the previous section, the correspondence of an LP "mode" with the HE and EH modes does not hold. Therefore, it is better to employ a new name for hybrid modes so as not to break the correspondence ($LP_{mn} - HE_{m+1n}$ and EH_{m-1n}). In this paper, hybrid modes are expressed by Hyb_{mn}, and the correspondence and the field configurations are indicated by HE^r_{mn}-type and EH^r_{mn}-type, whose polarization states and field configurations are similar to those of the HE_{mn} and EH_{mn} modes in a cylindrical dielectric rod.

It is proposed that the approximate factorization method be utilized for the designation of hybrid modes, and if the crossover exists between hybrid modes with the same order of azimuthal variation, the dispersion characteristics should be rigorously analyzed in the vicinity of the crossover, and hybrid modes be classified by the new name Hyb_{mn}.

V. Conclusions

It is shown that several classifications of hybrid modes cannot be utilized due to the crossover of dispersion characteristics between hybrid modes with the same azimuthal number. The crossing phenomenon is clarified by calculating the polarization states and the field configurations of hybrid modes at the vicinity of the crossover. It becomes evident that the phenomenon is caused by the coupling between the HE-type and EH-type hybrid modes, and the hybrid modes change remarkably their polarization states and field configurations with each other at the crossover. A new scheme for the classification of hybrid modes in cylindrical optical fibers is also proposed.

Acknowledgment

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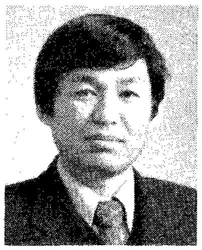

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Empirical Expressions for Fin-Line Design

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***Abstract*—This paper presents empirical expressions in closed form for the design of unilateral and bilateral fin-lines. The guided wavelength and the characteristic impedance calculated with these expressions agree, typically, within ± 2 percent with values obtained using numerical techniques in the normalized frequency range $0.35 \leqslant b/\lambda \leqslant 0.7$, which is suitable for most practical applications.**

I. Introduction

FIN-LINES FIND frequent applications in millimeter-wave integrated-circuit design. This is attributed to their favorable properties, such as low dispersion, broad single-mode bandwidth, moderate attenuation, and compatibility with semiconductor devices. Among various possible configurations, unilateral and bilateral fin-lines are of particular interest (see Fig. 1).

To this date, the propagation characteristics of fin-lines have been obtained with various methods. An early paper by Meier [1] described the propagating mode as a variation of the dominant mode in ridged waveguide. His procedure requires a test measurement to determine the equivalent dielectric constant of the fin-line structure. This is both expensive and time consuming. The analysis procedures by Saad and Begemann [2] and Hoefer [3] are based on ridged waveguide theory, and provide only an approximate solution. On the other hand, an accurate description of propagation in fin-lines, such as presented by Hofmann [4], and

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